1. Let $f : \mathbb{R} \to \mathbb{R}$ be any smooth function, and define $X = f(y)\frac{\partial}{\partial x}$ a vector field on $\mathbb{R}^2$. Find the flow for X.

2. Let $f : M \to N$ and $g : L \to M$ be smooth maps of manifolds. For a tensor $\tau \in \mathcal{T}^r(N)$, show that $(f \circ g)^*\tau = g^*(f^*\tau)$.

3. Given $f \in C^\infty(M)$, define $\tau_f(X, Y) = XYf$ for $X, Y \in \mathfrak{X}(M)$. Show that τ_f is bilinear in X and Y. Is τ_f a tensor field?

4. If you rescale a Riemannian metric, what effect does this have on distances between points?

   A Riemannian manifold (M, g) has a metric d giving the distance between any two points in M. For $\lambda > 0$, λg is a Riemannian metric on M, and say $(M, \lambda g)$ has metric d_λ. Express $d_\lambda(x, y)$ in terms of $d(x, y)$ and λ.

5. Let M be an m-manifold, and $x \in M$. Show there is a Riemannian metric g on M and a neighborhood U of x so that $(U, g|_U)$ is isometric to a subset of $\mathbb{R}^n$ with the Euclidean metric.

6. Define the one-form
$$\tau = xdy - ydx + zdw - wdz$$
   on the sphere $S^3 = \{(x, y, z, w)|x^2 + y^2 + z^2 + w^2 = 1\} \subset \mathbb{R}^4$. Fix $a, b \in \mathbb{R}$ with $a^2 + b^2 = 1$. For $t \in [0, 2\pi]$ define a curve $\gamma(t) = (a \cos t, a \sin t, b \cos t, b \sin t) \subset S^3$.

   Compute $\int_\gamma \tau$.

   Bonus generalization(optional, only slightly harder):

   If $a, b \in \mathbb{C}$ with $|a|^2 + |b|^2 = 1$ then $\gamma(t) = e^{it}(a, b)$ is a circle in $S^3 \subset \mathbb{C}^2$. Compute $\int_\gamma \tau$.