Math 452 - Homework 9 - Due Monday, April 19

Do Conway, Pg 185 # 2; Pg 194 # 2,3,4, and question 1 below.

1. Let $[x]$ be the greatest integer function, i.e. $[x]$ is the greatest integer less than or equal to x. Prove that for $\text{Re}(s) > 1$,

$$\zeta(s) = s \int_0^\infty [x]\, x^{-s} \frac{dx}{x}.$$

(So the zeta function is related to the Mellin transform of $[x]$.)

Hints

Con. 185 # 2: Use the Functional Equation (7.7) to write $\Gamma(1-z)$ in terms of $\Gamma(-z)$ and then expand out using the infinite product formula for Γ (7.3). On the other side, use the infinite product formula for $\sin(\pi z)$.

Con. 194 # 2: Assume $\sum p_n^{-1}$ actually converges. Use this to show that the Euler product for $\zeta(z)$ converges when $z = 1$, just by applying our usual convergence test for infinite products. But $\zeta(z)$ has a pole at $z = 1$.

Con. 194 # 3,4: Write out the left hand side as a product of infinite series via Def. 8.1. Then multiply the series.

Prob. 1: The value of $[x]$ is constant on the interval $[n, n+1)$. Split the integral into a sum of integrals, one for each n, integrate each one, and note that the resulting series is telescoping.