4/5/19 **Math 1520 – Quiz 8** Name: ____________________

(10) 1. Give an example of an alternating series which does not converge.

Solution: One example is $1 - 1 + 1 - 1 + 1 - 1 + 1 - \cdots$.

(10) 2. Let $a_n = \frac{n+1}{n!}$ and consider the series $\displaystyle\sum_{n=0}^{\infty} a_n = 1 + 2 + \frac{3}{2} + \frac{2}{3} + \frac{5}{24} + \cdots$.

Calculate the ratio $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ for the series. Does the series converge?

Solution:

$$r = \lim_{n\to\infty} \frac{n+2}{(n+1)!}\frac{n!}{n+1} = \lim_{n\to\infty} \frac{n+2}{(n+1)^2} = 0.$$

The series converges.

(10) 3. For which x does the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{5 \cdot 3^n}$ converge?

Solution: The series converges for $x \in (-3, 3)$.

Let $r = \lim_{n\to\infty} \left| \frac{x^{n+1}}{5 \cdot 3^{n+1}} \frac{5 \cdot 3^n}{x^n} \right| = \lim_{n\to\infty} \left| \frac{x}{3} \right| = \left| \frac{x}{3} \right|$. By the ratio test, the series converges when $r < 1$, which happens when $|x| < 3$. When $x = \pm 3$ the ratio test fails but it is easy to see the series diverges.