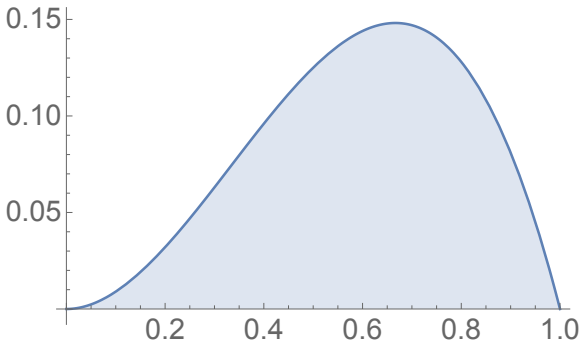
3/8/19 Math 1520 – Quiz 6 Name: _____

(15) 1. A flat sheet is bounded by $f(x) = x^2 - x^3$ and the x-axis as shown.

Find the center of mass $\bar{x}$.



Solution: The area is

$$A = \int_0^1 x^2 - x^3 dx = x^3/3 - x^4/4\Big|_0^1 = 1/3 - 1/4 = 1/12$$

The moment is

$$M_x = \int_0^1 x^3 - x^4 dx = x^4/4 - x^5/5\Big|_0^1 = 1/4 - 1/5 = 1/20$$

So that $\bar{x} = M_x/A = 3/5 = 0.6$

(15) 2. You want to dig a circular mine shaft with a 5 foot radius down to a depth of 400 feet. As you drill, you need to lift the rock debris out of the shaft. How much total work is required?

Assume the rock weighs 100 $^{\text{lb}}/_{\text{ft}^3}$

Solution: Consider a horizontal slice at depth y of thickness Δy. This has volume $25\pi\Delta y$ ft^3. It weighs $2500\pi\Delta y$ lb. To lift it out, you must move it up by y, so the work required for this slice is $y \cdot 2500\pi\Delta y$. Sum over all slices and let $\Delta y \to 0$ to get the total work:

$$\int_0^{400} 2500\pi \cdot y \; dy = 200000000\pi \approx 6.28 \times 10^8 \text{ ft-lb}$$