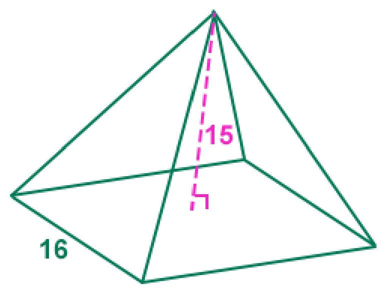
1. "Gabriel's Horn" is the shape you get by rotating the graph of $y = 1/x$ around the x-axis, for $x \in [1, \infty)$. Compute the volume of Gabriel's Horn.

Solution: Slice into disks with width Δx and radius $1/x$. Each disk has volume $\frac{\pi}{x^2}\Delta x$, so the volume of Gabriel's Horn is:

$$\sum \frac{\pi}{x^2}\Delta x \to \int_1^\infty \frac{\pi}{x^2}dx = -\frac{\pi}{x}\Big|_1^\infty = \pi.$$

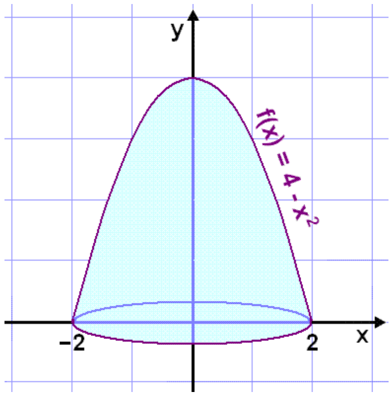
2. Compute the volume of pyramid whose base is a square of side length 16 and whose height is 15.



Solution: Cut into slices horizontally, and let t be the distance of the slice from the top. The slices are squares with thickness Δt. As t varies from 0 to 15, the side length of the square varies from 0 to 16. In terms of t, the square has side length $\frac{16}{15}t$, so each slice has volume $\left(\frac{16}{15}t\right)^2 \Delta t$. Then the pyramid's volume is given by

$$\sum \left(\frac{16}{15}t\right)^2 \Delta t \to \int_0^{15} \left(\frac{16}{15}t\right)^2 dt = \left(\frac{16}{15}\right)^2 \frac{t^3}{3}\Big|_0^{15} = \frac{16^2 \cdot 15}{3} = 1280.$$

3. Compute the volume of the parabaloid generated by rotation the graph of $f(x) = 4-x^2$ around the y-axis as shown.



Solution: Slice horizontally into disks. The disk at height y has radius x, where $y = 4 - x^2$, so that $x = \sqrt{4-y}$. The disk at height y has volume $\pi(\sqrt{4-y})^2 \Delta y = \pi(4-y)\Delta y$. The volume of the solid is then

$$\sum \pi(4-y)\Delta y \to \int_0^4 \pi(4-y)dy = \pi \left(4y - \frac{y^2}{2}\right)\Bigg|_0^4 = 8\pi$$