1. Does the series converge? Explain why or why not.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{n^2+1}$

   Solution: This converges by comparison with $\sum_{n=0}^{\infty} \frac{2}{n^2}$, since $\frac{2}{n^2+1} < \frac{2}{n^2}$ and $\sum_{n=0}^{\infty} \frac{1}{n^2}$ is a p-series with $p > 1$. You can also see this by the integral test, since $\int_0^\infty \frac{2}{x^2+1}dx = 2\arctan(x)\Big|_0^\infty = \pi < \infty$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$

   Solution: This diverges since $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2n} = \frac{1}{2}\sum_{n=0}^{\infty} \frac{1}{n}$, and the harmonic series diverges.

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 \log(n)}$

   Solution: This converges. Compare with $\sum_{n=2}^{\infty} \frac{1}{n^2}$, since $\frac{1}{n^2\log(n)} < \frac{1}{n^2}$.

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{e^n+n}{e^n+2^n}$

   Solution: This diverges since the terms $\frac{e^n+n}{e^n+2^n} \to 1$ (not zero!) as $n \to \infty$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n^3-1}{n^6+1}}$

   Solution: This converges. $\displaystyle\sqrt{\frac{n^3-1}{n^6+1}} < \sqrt{\frac{n^3}{n^6}} = \frac{1}{n^{3/2}}$. The series converges by comparison with the convergent series $\sum \frac{1}{n^{3/2}}$.

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 - 5n/2}$

   Solution: This converges. First, you want to ignore the $n = 1$ and $n = 2$ term since they are negative. Then, use the integral test:

   $$\int_3^\infty \frac{dx}{x^2-5x/2} = \frac{2}{5}\log\left(\frac{x-5/2}{x}\right)\Bigg|_3^\infty = \frac{2}{5}\log 6 < \infty.$$

   Alternately, you can use the comparison test. You'd like to compare with $\sum \frac{1}{n^2}$ but the terms of our series are larger than $\frac{1}{n^2}$. The trick is to notice that eventually $5n/2 < \frac{n^2}{2}$. This happens for all $n > 5$. Then $n^2 - 5n/2 > n^2 - \frac{n^2}{2} = n^2/2$ so that $\frac{1}{n^2-5n/2} < \frac{2}{n^2}$. The series converges since $\sum \frac{2}{n^2}$ converges.