

1. Write out the first five terms of these series as a sum. Compute the first five partial sums.

(a)  $\sum_{n=1}^{\infty} \frac{n}{2}$

**Solution:**  $1/2 + 2/2 + 3/2 + 4/2 + 5/2 + \dots$   
 Partial sums are 0.5, 1.5, 3, 5, 7.5.

(b)  $\sum_{n=0}^{\infty} \frac{3}{2^n}$

**Solution:**  $3 + 3/2 + 3/4 + 3/8 + 3/16 + \dots$   
 Partial sums are 3, 4.5, 5.25, 5.625, 5.8125.

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$

**Solution:**  $1 - 1/3 + 1/5 - 1/7 + 1/9 - \dots$   
 Partial sums are 1, 0.666..., 0.866..., 0.7238..., 0.8349....

2. Write these series using summation notation

(a)  $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \dots$

**Solution:**  $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(b)  $\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$

**Solution:**  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

(c)  $\frac{1}{e^3} + \frac{2}{e^5} + \frac{2^2}{e^7} + \frac{2^3}{e^9} + \frac{2^4}{e^{11}} + \dots$

**Solution:**  $\sum_{n=0}^{\infty} \frac{2^n}{e^{3+2n}}$

3. Find the sum of these series, if they converge:

(a)  $\sum_{n=2}^{\infty} \frac{1}{n^2 - n}$

**Solution:** Partial sums are  $1/2, 2/3, 3/4, 4/5, \dots$ , so the series converges to 1.

(b)  $\sum_{n=1}^{\infty} \frac{6}{10^n}$

**Solution:** Partial sums are 0.6, 0.66, 0.666, 0.6666, ..., so the series converges to  $2/3$ .

(c)  $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \dots$

**Solution:** This series is geometric with ratio  $r = 2/3$ , and converges to  $\frac{1}{1-2/3} = 3$ .

(d)  $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \frac{1}{13} + \frac{1}{17} + \dots$  (reciprocals of the primes)

**Solution:** It is quite hard to see that this series diverges. See [https://en.wikipedia.org/wiki/Divergence\\_of\\_the\\_sum\\_of\\_the\\_reciprocals\\_of\\_the\\_primes](https://en.wikipedia.org/wiki/Divergence_of_the_sum_of_the_reciprocals_of_the_primes)