1. Find the limit of each sequence, if it exists.

(a) $a_n = \dfrac{n^2+1}{2n^2+1}$

Solution: $1/2$

(b) $b_n = \cos\left(\dfrac{1}{n}\right)$

Solution: 1

(c) $c_n = \cos\left(\dfrac{\pi n}{2}\right)$

Solution: Limit does not exist.

(d) $d_n = \dfrac{n!}{4^n}$

Solution: Limit does not exist (it diverges to ∞).

(e) $e_1 = 1$ and $e_n = \cos(e_{n-1})$

Solution: $\approx 0.7390851\ldots$

(f) $f_n = n\sin(\dfrac{1}{n})$

Solution: 1

(g) $g_n = \dfrac{1}{\sin(n)}$

Solution: Limit does not exist.

(h) $h_1 = 2$ and $h_{n+1} = \dfrac{h_n^2+2}{2h_n}$

Solution: $\sqrt{2}$

(i) $k_n = n!\sin(\pi n)$

Solution: 0

(j) $\gamma_n = \displaystyle\int_1^n \frac{1}{x}dx - \left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}\right)$

Solution: This limit, known as the *Euler-Mascheroni constant* is approximately 0.57721.