

Carbon-14 is a radioactive isotope of carbon that decays over time into the stable and much more common carbon-12.

When a plant or animal is alive, it exchanges carbon with the atmosphere and so has the same percentage of carbon-14 as the atmosphere. When the plant or animal dies, it stops taking in carbon-14 from the atmosphere, and the carbon-14 it does contain starts to decay, so over time it has less carbon-14 than the atmosphere.

We can use this to figure out the age of artifacts by estimating the original mass of carbon-14 in the object and the amount at present.¹

1. Let $P(t)$ be the amount of Carbon-14 after t years. With radioactive decay, the rate of change of P is proportional to P , so we write the differential equation:

$$\frac{dP}{dt} = rP$$

Find the solution to the decay equation in terms of the initial amount P_0 of carbon-14.

2. Carbon-14 has a half life of 5730 years. Use this to find the rate of decay r .

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| Solution: $r = \frac{\log(2)}{5730} \approx 0.00012$ |
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3. A bowl made of oak has about 40% of the carbon-14 that a similar quantity of living oak has today. Estimate the age of the bowl.

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| Solution: $t = -\log(0.4)/r \approx 7575$ years. |
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¹Willard Libby won the Nobel Prize in Chemistry in 1960 for this idea.