1. Sketch the curve given by $x = t^2, y = t^3$ for $t \in [0, 2]$. Find its arclength exactly.

Solution: The arclength is:

$$\int_0^2 \sqrt{(2t)^2 + (3t^2)^2}dt = \int_0^2 t\sqrt{4 + 9t^2}dt$$

Now let $u = 4 + 9t^2$, so $du/18 = tdt$, and get

$$\frac{1}{18}\int_4^{40} \sqrt{u}du = \frac{1}{18} \cdot \frac{2}{3}u^{3/2}\Big|_4^{40} = \frac{1}{27}(40^{3/2} - 4^{3/2}) = \frac{8}{27}(\sqrt{1000} - 1) \approx 9.07.$$

2. The parameterized curve $(3\sin t, \cos t)$ gives an ellipse, with t running from 0 to 2π. Sketch the curve and then find the circumference of the ellipse.

Solution: The arclength is:

$$\int_0^{2\pi} \sqrt{(3\cos t)^2 + (-\sin t)^2}dt = \int_0^{2\pi} \sqrt{9\cos^2 t + \sin^2 t}dt$$

The integral is not elementary (it's called an *elliptic integral*) and evaluates numerically to about 13.36.