1. $\int_0^\infty xe^{-x^2}\ dx$

Solution: Substitute $u = x^2$ to get:

$$\int_0^\infty xe^{-x^2}dx = \frac{1}{2}\int_0^\infty e^{-u}du = \lim_{b\to\infty}\frac{1}{2}\left(-e^{-b}+e^{-0}\right) = \frac{1}{2}$$

2. $\int_{-\infty}^0 \frac{4}{x^2-6x+9}\ dx$

Solution:

$$\int_{-\infty}^0 \frac{4}{x^2-6x+9}dx = 4\int_{-\infty}^0 \frac{dx}{(x-3)^2} = \lim_{a\to-\infty} -\frac{4}{x-3}\bigg|_a^0 = \lim_{a\to-\infty}\frac{4}{3}+\frac{4}{a-3} = \frac{4}{3}$$

3. $\int_e^\infty \frac{1}{x\log(x)}\ dx$

Solution: Substitute $u = \log x$ to get

$$\int_e^\infty \frac{1}{x\log(x)}dx = \lim_{b\to\infty}\int_e^b \frac{1}{x\log(x)}dx = \lim_{b\to\infty}\int_1^{\log(b)}\frac{1}{u}du = \lim_{b\to\infty}\log(\log(b)) - \log(1) = \infty.$$

This integral diverges, but incredibly slowly: log(log(100000000000000000000)) is only 3.83.

4. $\int_1^\infty \frac{1}{\sqrt{x}}\ dx$

Solution: This diverges since

$$\int_1^\infty \frac{1}{\sqrt{x}}dx = \lim_{b\to\infty} 2\sqrt{x}\bigg|_1^b = \infty$$

5. $\int_0^\infty \frac{1}{x^2+3x+2}\ dx$

Solution: Using partial fractions,

$$\int_0^\infty \frac{1}{x^2+3x+2}dx = \int_0^\infty \frac{1}{x+1}-\frac{1}{x+2}dx = \lim_{b\to\infty}\log(x+1)-\log(x+2)\bigg|_0^b$$

$$= \lim_{b\to\infty}\log\left(\frac{x+1}{x+2}\right)\bigg|_0^b = \log(1)-\log(\frac{1}{2}) = \log(2) \approx 0.693$$