Fourier Series

The Fourier series for $f(x)$ defined on $[-\pi, \pi]$ is

$$f(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + a_4 \cos(4x) + \cdots$$
$$+ b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + b_4 \sin(4x) + \cdots$$

where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)dx$$

Note that for even functions, only cosine terms are non-zero, and for odd functions only sine terms are non-zero.

Let $f(x) = x^2$ on the interval $[-\pi, \pi]$. Since f is even, the Fourier series for $f(x)$ will have only a_0 and cosine terms.

1. Find the value of a_0.

2. Find the value of a_n for $n > 0$. Use scratch paper and/or get computer assistance.

3. Write the Fourier series for x^2 on $[-\pi, \pi]$:

   $x^2 =$

4. Graph the first few terms of the Fourier series and see how it converges to x^2.

5. Solve the "Basel Problem": Find the value of the series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \cdots$ To do this, plug in $x = \pi$ to your equation in problem 3 and simplify.