Error Bounds

We want to understand the convergence of the series

$$f(x) = \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

Let $P_n(x)$ be the n^{th} Taylor polynomial for $\sin(x)$, and $E_n(x) = |f(x) - P_n(x)|$ be the error in approximating $f(x)$ with $P_n(x)$.

1. Let's look at $x = 0.5$. What is $\sin(0.5)$?

   Fill out the following table:

n	0	1	2	3	4	5
$P_n(x)$						
$P_n(0.5)$						
$\|E_n(0.5)\|$						
$\frac{0.5^{n+1}}{(n+1)!}$						

2. Does $\frac{0.5^{n+1}}{(n+1)!}$ seem to be a good estimate for the error in the approximation?