1. Sketch the triangle formed by $y = \frac{1}{2}x$, $x = 3$ and the x-axis. Find its centroid.

Solution:

The area is $\frac{1}{2}bh = \frac{1}{2}3 \cdot \frac{3}{2} = \frac{9}{4}$.

$$M_x = \int_0^3 x \cdot \tfrac{1}{2}x\,dx = \left.\frac{x^3}{6}\right|_0^3 = \frac{9}{2}$$

so $\bar{x} = 2$. For the y moment, y slices have width $3 - 2y$, so

$$M_y = \int_0^{3/2} y \cdot (3 - 2y)\,dy = \left.\frac{3y^2}{2} - \frac{2y^3}{3}\right|_0^{3/2} = \frac{27}{8} - \frac{9}{4} = \frac{9}{8}$$

so $\bar{y} = \frac{1}{2}$ and the centroid is $(2, \frac{1}{2})$.
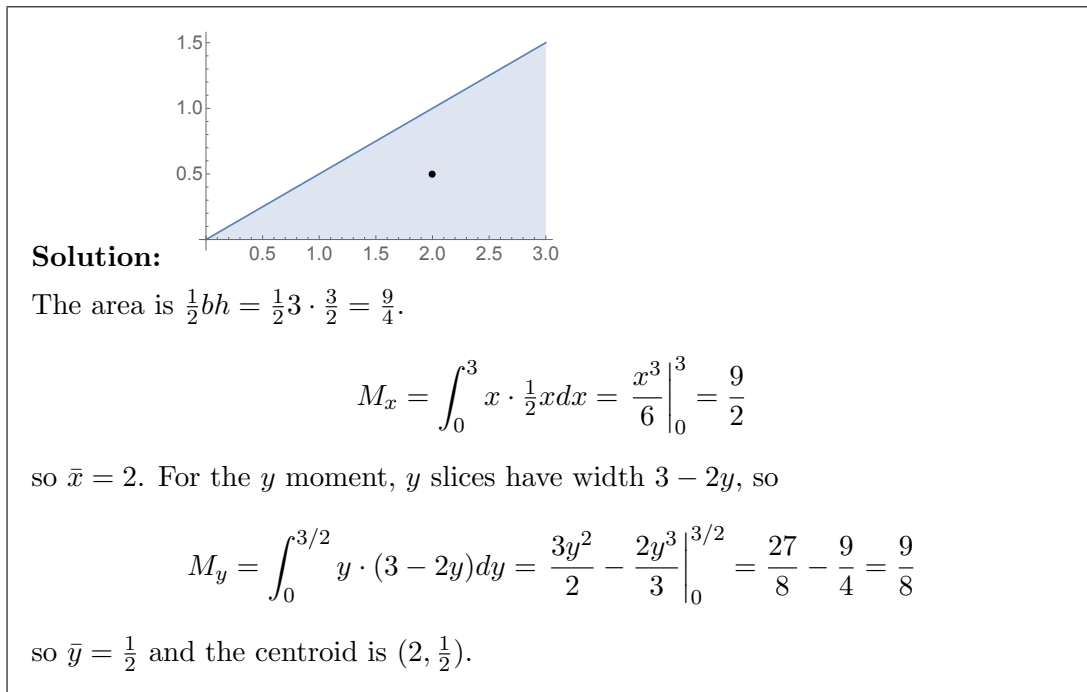
2. A solid pyramid has a square base of side length 5 and a height of 40. How high is the center of mass?

Solution: Slice horizontally into squares. The square at height h has side length $s = 5 - \frac{h}{8}$. Then

$$\text{Volume} = \int_0^{40} (5 - \tfrac{h}{8})^2\,dh = \int_5^0 s^2(-8ds) = 8\int_0^5 s^2\,ds = \left.\frac{8s^3}{3}\right|_0^5 = \frac{1000}{3}$$

$$M_h = \int_0^{40} h \cdot (5 - \tfrac{h}{8})^2\,dh = 8\int_0^5 (40 - 8s) \cdot s^2\,ds = 8\left.\left(\frac{40s^3}{3} - 2s^4\right)\right|_0^5 = \frac{10000}{3}$$

Then $\bar{h} = M_h/\text{Volume} = 10$. The center of mass is at height 10, one quarter of the height of the pyramid.