Taylor Series for $\arctan(x)$

1. Let $f(x) = \arctan(x)$. What is the derivative $f'(x)$?

Solution:

$$f'(x) = \frac{1}{1+x^2}$$

2. Use the geometric series to get the series for $f'(x)$ at $x = 0$.

Solution:

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \cdots$$

3. Integrate your series term-by-term to get a series for $\arctan(x)$ at $x = 0$. Check that the constant term is correct by plugging in $x = 0$.

Solution:

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \frac{x^{11}}{11} + \cdots$$

Both sides are 0 when $x = 0$, so there is no $+C$.

4. What is the interval of convergence of the series for $\arctan(x)$?

Solution: The ratio test shows the radius of convergence is 1. At both $x = 1$ and $x = -1$, the series converges by the alternating series test. The interval of convergence is $[-1, 1]$.

5. Plot on the same graph both $f(x)$ and the 9th degree Taylor polynomial for f.

6. Plug in $x = 1$ to find the Leibniz series formula for π.

Solution:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$