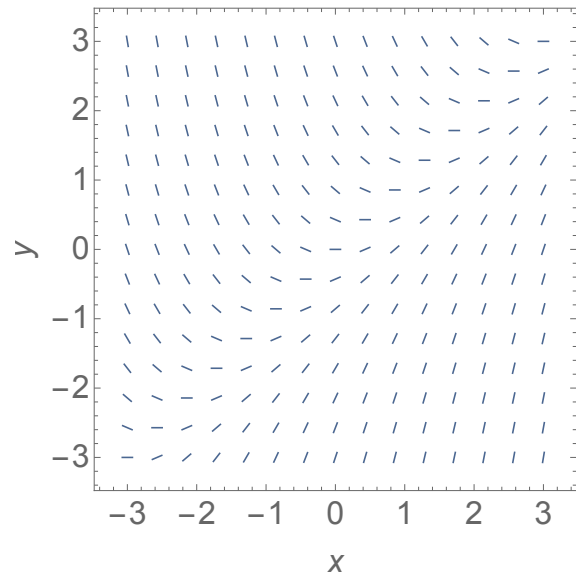


## Math 1520 – Sample Final Exam

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration (TI-89, for example).

There are 15 questions, worth a total of 150 points.

- (10) 1. Find the solution to the separable differential equation  $\frac{dy}{dx} = x\sqrt{y}$  with initial condition  $y = 4$  when  $x = 0$ .
- (10) 2. The slope field for the differential equation  $\frac{dy}{dx} = x - y$  is shown below.
- (a) Sketch the two solutions which have initial conditions  $(-3, 3)$  and  $(0, -3)$ .
- (b) Guess one linear solution to the differential equation and check that it works.



- (10) 3. The Pioneer 10 spacecraft is powered by plutonium radio-thermal generators. The power produced depends directly on the amount of plutonium remaining. The amount of plutonium,  $P$ , decays according to the differential equation  $\frac{dP}{dt} = -rP$ .
- (a) Find  $r$ , the decay coefficient, given that the half-life of plutonium is 87.72 years.
- (b) At launch, the power generated was 2580 Watts. How much power was being generated when it sent its last signal in 2003, 31 years after launch?
- (10) 4. The (infinite) region bounded by the curve  $y = e^{-x}$ , the positive  $y$ -axis and the positive  $x$ -axis is revolved around the  $x$ -axis. Find the volume of this solid of revolution.
- (10) 5. Integrate  $\int \frac{dx}{x^2 + 2x}$
- (10) 6. Integrate  $\int \frac{dx}{x^2 + 2x + 1}$

(10) 7. Integrate  $\int \frac{dx}{x^2 + 2x + 2}$

(10) 8.  $2e$  or not  $2e$ ? For each, decide if the value is  $2e$  or not  $2e$ .

(a)  $\sum_{n=0}^{\infty} \frac{2}{n!}$  (a) \_\_\_\_\_

(b)  $\lim_{n \rightarrow \infty} 2 \left(1 + \frac{1}{n}\right)^n$  (b) \_\_\_\_\_

(c)  $\int_2^{\infty} \frac{dx}{x}$  (c) \_\_\_\_\_

(d)  $\sum_{n=0}^{\infty} \frac{e}{2^n}$  (d) \_\_\_\_\_

(e)  $4 + \frac{4}{2 + \frac{3}{3 + \frac{4}{4 + \frac{5}{5 + \frac{6}{6 + \frac{7}{\ddots}}}}}}$  (e) \_\_\_\_\_

(f)  $\lim_{n \rightarrow \infty} \frac{2n}{\sqrt[n]{n!}}$  (f) \_\_\_\_\_

(g)  $\sum_{n=0}^{\infty} \frac{n+1}{n!}$  (g) \_\_\_\_\_

(10) 9. Find the sum of the geometric series  $\frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \frac{5}{81} + \frac{5}{243} + \dots$ .

(10) 10. Define a sequence by  $a_0 = 1$ ,  $a_n = \frac{1+a_{n-1}}{2+a_{n-1}}$ . Write out the first five terms of this sequence. Bonus: What can you say about the limit of this sequence?

(10) 11. Match the description to the series:

- |   |                       |
|---|-----------------------|
| (a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$                            | 1. Power series       |
| (b) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots$                             | 2. Harmonic series    |
| (c) $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \dots$ | 3. Geometric series   |
| (d) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$                              | 4. Alternating series |
| (e) Giants beat Rangers in 5 games.   | 5. World series       |

(10) 12. The Fourier series for  $f(x) = x$  on the interval  $[-\pi, \pi]$  is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin(nx).$$

where

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin(nx) dx.$$

Compute the coefficients  $b_n$  and write the first five terms of the Fourier series for  $f$ .

(10) 13. Find the Taylor series for  $f(x) = e^x$  at the point  $x = 1$ . Write using summation notation or show at least five terms.

(10) 14. Find the fifth derivative of  $f(x) = \frac{x}{1-x^2}$  at  $x = 0$ .

(10) 15. Give an example of a power series centered at 3 with radius of convergence equal to 5. Does your series converge at  $x = 0$ ? Does it converge at  $x = 8$ ?