3/29/19 **Math 1520 – Exam 2** Name: ____________________

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration (TI-89, for example).

There are 10 questions, worth a total of 100 points.

(10) 1. (a) Define a sequence by $a_1 = 1$ and $a_{n+1} = a_n^2 - 2a_n$. Calculate the first six terms of the sequence.

(b) What is $\lim_{n\to\infty} a_n$?

Solution: The first six terms are 1, -1, 3, 3, 3, 3. The sequence continues with 3 forever, so $\lim_{n\to\infty} a_n = 3$.

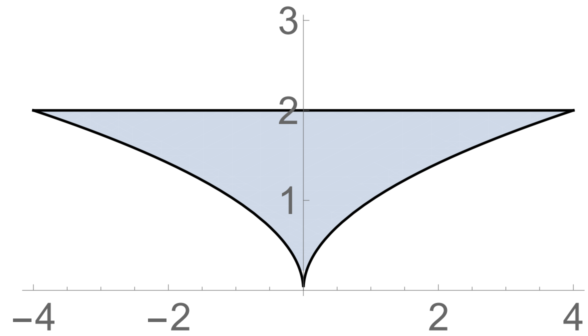
(10) 2. Find the sum of the infinite geometric series $10 - 2 + \frac{2}{5} - \frac{2}{25} + \frac{2}{125} - \frac{2}{625} + \cdots$

Solution:

$$10 - 2 + \frac{2}{5} - \frac{2}{25} + \frac{2}{125} - \frac{2}{625} + \cdots = 10\left(1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \frac{2}{625} - \cdots\right) \quad (1)$$

$$= \frac{10}{1-(-1/5)} = \frac{25}{3} \quad (2)$$

(10) 3. Find the area of the region shown, which has boundary $y = \sqrt{x}$, $y = \sqrt{-x}$, $y = 2$.



Solution: Slicing horizontally, the area is given by

$$\int_0^2 2y^2\, dy = \frac{2}{3}y^3\Big|_0^2 = \frac{16}{3}$$

(10) 4. Calculate the length of the parametric curve $x = t\sin(t) + \cos(t)$, $y = \sin(t) - t\cos(t)$ from $t = 0$ to $t = 10$.

Solution:

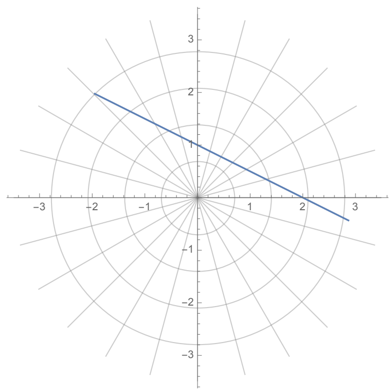
$$\frac{dx}{dt} = t\cos(t), \quad \frac{dy}{dt} = t\sin(t)$$

so the length is

$$\int_0^{10} \sqrt{t^2\cos^2(t) + t^2\sin^2(t)}\, dt = \int_0^{10} \sqrt{t^2}\, dt = \int_0^{10} t\, dt = 50.$$

(10) 5. The polar equation $r = \dfrac{2}{2\sin(\theta) + \cos(\theta)}$ describes a straight line.

Accurately sketch the line, then give its x-intercept, y-intercept, and slope.



Solution:

The x intercept is 2, the y intercept is 1, and the line has slope $-1/2$.

(10) 6. Use the integral test to decide if the series $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$ converges. Show work.

Solution:

$$\int_3^{\infty} \frac{1}{x\ln(x)}dx = \ln(\ln(x))|_3^{\infty} = \infty$$

Since the integral diverges, the series does too.

(10) 7. (a) Give an example of one sequence that satisfies all three of these conditions:

- It is not decreasing
- It is not increasing
- It converges

Solution: A simple example is $1, 0, \frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}, 0, \ldots$ which converges to 0.

(b) Give an example of one sequence that satisfies all three of these conditions:

- It is not decreasing
- It is not increasing
- It diverges

Solution: A simple example is $1, 0, 2, 0, 3, 0, 4, 0, \ldots$ which does not converge.

(10) 8. For each series, decide if it converges or diverges (no explanation required).

(a) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots$

(b) $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \cdots$

(c) $\sum_{n=1}^{\infty} \frac{2019}{n^2 + \sqrt{7}}$

(d) $\sum_{n=1}^{\infty} n^{-\frac{2}{3}}$

(e) $\sum_{n=1}^{\infty} \ln(\sqrt{n})$

Solution:

(a) Diverges. This is the harmonic series.

(b) Converges (to 2/3). This is a geometric series with common ratio $-1/2$.

(c) Converges. Since $n^2 + \sqrt{7} > n^2$,

$$\sum_{n=1}^{\infty} \frac{2019}{n^2 + \sqrt{7}} < 2019 \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

The series converges by the comparison test, since $\sum 1/n^2$ converges.

(d) Diverges. This is a p series with $p = \frac{2}{3} < 1$. Or use the integral test with $\int_1^\infty x^{-2/3} dx = \infty$.

(e) Diverges. The terms don't go to zero. As $n \to \infty$, $\ln(\sqrt{n}) \to \infty$.

(10) 9. Let R be the region bounded by $y = e^{-x}$, $x = 0$, $x = 3$ and the x-axis, and let S be the solid formed by rotating R around the x-axis.

Find the volume of S.

Solution:

$$V = \int_0^3 \pi(e^{-x})^2 dx = \pi \int_0^3 e^{-2x} dx = \frac{\pi}{-2} e^{-2x} \Big|_0^3 = \frac{\pi}{2}(1 - e^{-6}) \approx 1.5669.$$

(10) 10. Find the centroid of the solid S from question 9. By symmetry, you only need to find $\bar{x}$.

Solution: The x moment is

$$M_x = \int_0^3 x\pi(e^{-x})^2 dx \quad \text{(now integrate by parts)} \tag{3}$$

$$= -\frac{\pi}{2} x e^{-2x} \Big|_0^3 + \frac{\pi}{2} \int_0^3 e^{-2x} dx \tag{4}$$

$$= -\frac{3\pi}{2} e^{-6} - \frac{\pi}{4} e^{-2x} \Big|_0^3 \tag{5}$$

$$= -\frac{3\pi}{2} e^{-6} - \frac{\pi}{4} e^{-6} + \frac{\pi}{4} \tag{6}$$

$$= \frac{\pi}{4}\left(1 - 7e^{-6}\right) \approx 0.7718 \tag{7}$$

Then $\bar{x} = \dfrac{M_x}{V} = \dfrac{1 - 7e^{-6}}{2 - 2e^{-6}} \approx 0.4925$