Fall 2018

Math 1520 – Sample Exam 2

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration.

There are 10 questions, worth a total of 100 points.

(10) 1. Let A be the region bounded by $y = x^4$, the x-axis, and the line $x = 1$. Find the volume of the solid obtained by rotating A around the x-axis.

(10) 2. Find the centroid of the region bounded by $y = \frac{1}{x}$ and the lines $y = \frac{1}{2}$ and $x = \frac{1}{2}$.

(10) 3. The functions $x(t) = a\cos(t)$ and $y(t) = b\sin(t)$ define an ellipse, for $0 \le t \le 2\pi$.

Set up an integral to find the circumference (arclength) of this ellipse.

You do not need to work out the integral. It is called an *elliptic integral* and cannot be computed, except numerically.

(10) 4. Sketch an accurate plot of the curve given in polar coordinates by $r = \sin(2\theta)$ for $\theta \in [0, 2\pi]$. (Hint: it looks like a clover with four petals)

(10) 5. Find the area of one petal of the curve in question 4. (You probably don't want to do this integral by hand, but this is a sample exam so it's ok.)

(10) 6. Give an example of a sequence $\{a_n\}$ which has these three properties:

- All terms are positive ($a_n > 0$ for all n).
- The sequence is not bounded above.
- The sequence $\frac{1}{a_n}$ does not converge.

(10) 7. Let $a_n = e^{\left(\frac{n-1}{n+1}\right)}$ for $n \ge 1$. Does the sequence $\{a_n\}$ converge or diverge? If it converges, find its limit.

(10) 8. Write the series

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$$

using summation notation (use a $\sum$). Then compute its value.

(10) 9. For each series, decide if it converges or diverges (no explanation required).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{10n^2 - n - 3}{n^4 - n^3 - n^2 + 2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{1}{n}}$

(c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots$

(d) $1000 + \frac{1000}{2} + \frac{1000}{4} + \frac{1000}{8} + \frac{1000}{16} + \frac{1000}{32} + \cdots$

(e) $\sum_{n=0}^{\infty} \frac{\pi^n}{e^n}$

(10) 10. Find the first five partial sums of the series $\sum_{n=1}^{\infty} \frac{n}{10^n}$.