2/15/19 # Math 1520 – Exam 1 Name: ________________

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration (TI-89, for example).

There are 9 questions, worth a total of 100 points.

(20) 1. Do any two of these integrals (do not do all three - only your first two will be graded):

$$\text{A} = \int (1-25x^2)^{-3/2}\ dx \qquad \text{B} = \int x^2\sin(2x)\ dx \qquad \text{C} = \int \frac{27}{x^2+11x+10}\ dx$$

Solution:

(a) Use $x = \frac{1}{5}\sin(\theta)$ to get

$$\begin{aligned}\int (1-25x^2)^{-3/2}\ dx &= \int (1-\sin^2(\theta))^{-3/2}\cdot\frac{1}{5}\cos(\theta)d\theta \\ &= \frac{1}{5}\int \frac{d\theta}{\cos^2(\theta)} \\ &= \frac{1}{5}\int \sec^2(\theta)d\theta = \frac{1}{5}\tan(\theta)+C \\ &= \frac{x}{\sqrt{1-25x^2}}+C\end{aligned}$$

(b)

$$\begin{aligned}\int x^2\sin(2x)\ dx &= -\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)dx \\ &= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) - \int \frac{1}{2}\sin(2x)dx \\ &= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C \\ &= \left(\frac{1}{4}-\frac{x^2}{2}\right)\cos(2x) + \frac{x}{2}\sin(2x) + C.\end{aligned}$$

(c)

$$\begin{aligned}\int \frac{27}{x^2+11x+10}\ dx \int \frac{3}{x+1} - \frac{3}{x+10}dx \\ &= 3\ln|x+1| - 3\ln|x+10| + C \\ &= 3\ln\left|\frac{x+1}{x+10}\right| + C.\end{aligned}$$

(10) 2. Suppose $f(x)$ is a function given by the table below:

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	1.0	1.2	1.4	1.7	2.1	2.5	3.0	3.6	4.3	5.2

Give the best approximation you can to $\int_1^9 \frac{f(\sqrt{x})}{\sqrt{x}} dx$.

Solution: Set $u = \sqrt{x}$ so

$$\int_1^9 \frac{f(\sqrt{x})}{\sqrt{x}} dx = 2\int_1^3 f(u)du.$$

For this u integral, LEFT(2) $= 1.2 + 1.4 = 2.6$, RIGHT(2) $= 1.4 + 1.7 = 3.1$, and TRAP(2) $= 2.85$, so the best approximation is $2 *$ TRAP(2) $= 5.7$.

(10) 3. Sketch $f(x) = x^2$ on the interval $[-1, 3]$ and the area corresponding to LEFT(4).

(10) 4. Integrate $\int \frac{1}{x^2+4}\, dx$.

Solution: $\frac{1}{2}\arctan(\frac{x}{2}) + C$

(10) 5. Integrate $\int \frac{x}{x^2+4}\, dx$.

Solution: $\frac{1}{2}\ln(x^2+4) + C$.

(10) 6. Integrate $\int \frac{x^2}{x^2+4}\, dx$.

Solution: $x - 2\arctan(\frac{x}{2})$.

(10) 7. (a) Use the FTC to compute the exact value of the integral $\mathcal{I} = \int_1^5 \frac{dx}{x}$

> **Solution:** $\mathcal{I} = \ln(5) \approx 1.609$

(b) Compute these approximations to the integral $\mathcal{I}$ exactly or to three decimal places.

LEFT(2)	$8/3 \approx 2.666$
RIGHT(2)	$16/15 \approx 1.067$
TRAP(2)	$28/15 \approx 1.867$
MID(2)	$3/2 = 1.5$
SIMP(2)	$73/45 \approx 1.622$

(10) 8. Compute the exact value of the improper integral $\int_2^\infty e^{-3x} dx$.

> **Solution:**
>
> $$\int_2^\infty e^{-3x} dx = \lim_{b\to\infty} -\frac{1}{3}e^{-3x}\Big|_2^b = \lim_{b\to\infty} -\frac{1}{3}e^{-3b} + \frac{1}{3}e^{-6} = \frac{1}{3}e^{-6}$$

(10) 9. (a) Let $n \geq 1$. Show that $\displaystyle\int_0^1 (-\ln(x))^n \, dx = n \int_0^1 (-\ln(x))^{n-1} \, dx$.

> **Solution:** Use integration by parts with $u = (-\ln(x))^n$ and $dv = dx$. Then $du = n(-\ln(x))^{n-1} \cdot -\frac{1}{x}$ and $v = x$. So
>
> $$\int_0^1 (-\ln(x))^n \, dx = \lim_{a \to 0^+} x(-\ln(x))^n \Big|_a^1 + n \int_0^1 (-\ln(x))^{n-1} \, dx$$
> $$= n \int_0^1 (-\ln(x))^{n-1} \, dx.$$
>
> We used the fact that $\displaystyle\lim_{a \to 0^+} a(-\ln(a))^n = 0$.

(b) Calculate $\displaystyle\int_0^1 (-\ln(x))^5 \, dx$.

> **Solution:** Using part (a) repeatedly,
>
> $$\int_0^1 (-\ln(x))^5 \, dx = 5 \int_0^1 (-\ln(x))^4 \, dx = 5 \cdot 4 \int_0^1 (-\ln(x))^3 \, dx =$$
> $$= 5 \cdot 4 \cdot 3 \int_0^1 (-\ln(x))^2 \, dx = 5 \cdot 4 \cdot 3 \cdot 2 \int_0^1 (-\ln(x))^1 \, dx =$$
> $$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \int_0^1 1 \, dx = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$$