

Math 1520 – Sample Exam 1

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration (TI-89, for example).

There are 7 questions, worth a total of 100 points.

- (40) 1. Calculate four of these integrals (do not do all five - only your first four will be graded):

$$A = \int \frac{x^2 + x + 1}{x + 1} dx; \quad B = \int \frac{\sqrt{\ln(x+1)}}{x+1} dx; \quad C = \int x \cos(3x) dx$$

$$D = \int \frac{e^x}{1 + e^{2x}} dx; \quad E = \int \frac{1}{x^2 + 5x + 6} dx$$

- (10) 2. True or false:

(a) $\int \frac{1}{x} dx = x^0 + C$

(b) $\int \frac{1}{x} dx = x^{-1} + C$

(c) $\int \frac{1}{x} dx = \ln|x| + C$

(d) $\int \frac{1}{x} dx = \ln|2x| + C$

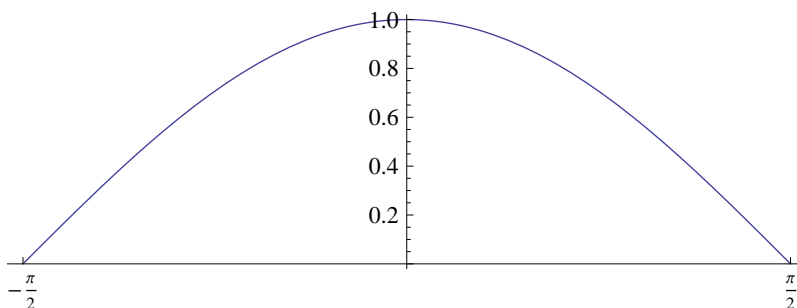
(e) $\int \frac{1}{x} dx = x \ln|x| - x + C$

- (10) 3. Let $\text{Li}(x) = \int \frac{dx}{\ln(x)}$. Calculate $\int \ln(\ln(x)) dx$ in terms of $\text{Li}(x)$.

- (10) 4. Compute $\int_2^\infty \frac{3}{(x-1)^{3/2}} dx$, if the integral converges.

- (10) 5. For which values of p does the integral $\int_0^\infty \frac{dx}{\sqrt{x^p + 1}}$ converge? Justify your answer.

- (10) 6. The picture below shows the graph of $\cos(x)$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



On the picture, draw rectangles corresponding to the left approximation LEFT(6).

Will LEFT(6) over- or under-approximate the area under the curve? (Look carefully and think!)

- (10) 7. (a) Compute $I = \int_0^1 x^2 dx$.
- (b) Compute the trapezoid approximation TRAP(2) for I .
- (c) Compute the midpoint approximation MID(2) for I .
- (d) Compute the Simpson's approximation $\text{SIMP}(2) = \frac{2\text{MID}(2) + \text{TRAP}(2)}{3}$
- (e) Explain why SIMP(2) gives the exact value of the integral I .