Math 1520 – Sample Exam 1

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration (TI-89, for example).

There are 7 questions, worth a total of 100 points.

(40) 1. Calculate four of these integrals (do not do all five - only your first four will be graded):

$$A = \int \frac{x^2+x+1}{x+1}dx; \qquad B = \int \frac{\sqrt{\ln(x+1)}}{x+1}dx; \qquad C = \int x\cos(3x)dx$$

$$D = \int \frac{e^x}{1+e^{2x}}dx; \qquad E = \int \frac{1}{x^2+5x+6}dx$$

Solution:

$$A = \int x + \frac{1}{x+1}dx = x^2/2 + \ln(x+1). \quad (1)$$

$$B = \int \sqrt{u}du = \frac{2}{3}(\ln(x+1))^{3/2}, \text{where } u = \ln(x+1). \quad (2)$$

$$C = \frac{x}{3}\sin(3x) + \frac{1}{9}\cos(3x). \quad (3)$$

$$D = \int \frac{du}{1+u^2} = \arctan(e^x), \text{where } u = e^x. \quad (4)$$

$$E = \int \frac{1}{x+2} - \frac{1}{x+3}dx = \ln|x+2| - \ln|x+3|. \quad (5)$$

(10) 2. True or false:

(a) $\int \frac{1}{x}dx = x^0 + C$

(b) $\int \frac{1}{x}dx = x^{-1} + C$

(c) $\int \frac{1}{x}dx = \ln|x| + C$

(d) $\int \frac{1}{x}dx = \ln|2x| + C$

(e) $\int \frac{1}{x}dx = x\ln|x| - x + C$

Solution: a. F; b. F; c. T; d. T; e. F

(10) 3. Let $\mathrm{Li}(x) = \int \frac{dx}{\ln(x)}$. Calculate $\int \ln(\ln(x))dx$ in terms of $\mathrm{Li}(x)$.

Solution: Integrate by parts with $u = \ln(\ln(x))$ and $dv = dx$. Then

$$\int \ln(\ln(x))dx = x\ln(\ln(x)) - \mathrm{Li}(x)$$

(10) 4. Compute $\int_2^\infty \frac{3}{(x-1)^{3/2}}\,dx$, if the integral converges.

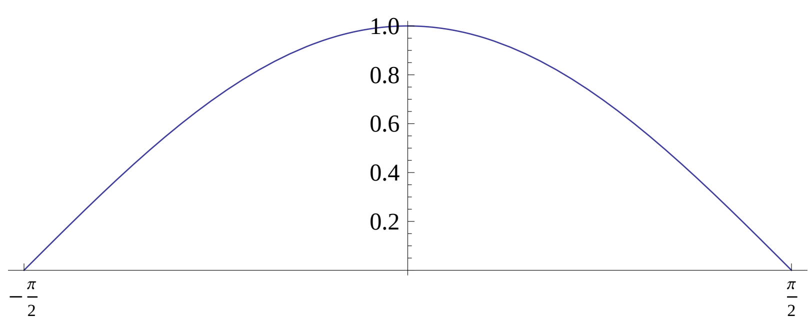
Solution:

$$\int_2^\infty \frac{3}{(x-1)^{3/2}}\,dx = \lim_{b\to\infty} \frac{-6}{\sqrt{b-1}} + 6 = 6.$$

(10) 5. For which values of p does the integral $\int_0^\infty \frac{dx}{\sqrt{x^p+1}}$ converge? Justify your answer.

Solution: Compare with $1/x^{p/2}$ to see that we need $p > 2$.

(10) 6. The picture below shows the graph of $\cos(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.



On the picture, draw rectangles corresponding to the left approximation LEFT(6).

Will LEFT(6) over- or under-approximate the area under the curve? (Look carefully and think!)

Solution: By the symmetry, LEFT(6) = RIGHT(6) for this picture, so LEFT(6) = TRAP(6), which it's easy to see is an underestimate since the curve is concave down.

(10) 7. (a) Compute $I = \int_0^1 x^2 dx$.

(b) Compute the trapezoid approximation TRAP(2) for I.

(c) Compute the midpoint approximation MID(2) for I.

(d) Compute the Simpson's approximation SIMP(2) $= \frac{2\,\text{MID}(2)+\text{TRAP}(2)}{3}$

(e) Explain why SIMP(2) gives the exact value of the integral I.

Solution: a) $I = 1/3$. b) TRAP(2) $= 3/8$. c) MID(2) $= 5/16$. d) SIMP(2) $= 1/3$. e) Because Simpson's rule approximates the curve with a quadratic function. Since the curve x^2 *is* quadratic, the approximation is exact.