Graduate students taking STAT 5088 should complete some "grad problems" over the course of the semester. I expect there will be around 5-8 of these problems, and you'll need to do a good job on half of them.

The beta distribution is defined by the density function:

$$f(t) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} t^{\alpha-1}(1-t)^{\beta-1}$$

for $t \in [0,1]$ and shape parameters $\alpha > 0$, $\beta > 0$.

The goal of this problem is to show that f defines a probability density function. Since f is clearly non-negative, the only thing to show is that $\int_0^1 f(t)dt = 1$.

Here is some help. Begin with the definition of the Gamma function and write:

$$\Gamma(\alpha)\Gamma(\beta) = \int_0^\infty x^{\alpha-1}e^{-x}dx \int_0^\infty y^{\beta-1}e^{-y}dy.$$

Next, the clever bit: combine into a double integral and change variables $x = st$, $y = s(1-t)$. Split back into an s integral times a t integral and rearrange terms to get the result.