Graduate students taking STAT 5088 should complete some "grad problems" over the course of the semester. I expect there will be around 5-8 of these problems, and you'll need to do a good job on half of them.

The goal of this problem is to implement a Metropolis-Hastings sampler for two parameters given a (non-normalized) PDF f(x, y).

In the one parameter sampler we implemented in the group lab, we kept a current parameter value t and updated it at each step by adding a normal rv. For a two parameter sampler, you need to keep track of two values s, t and at each step, your proposal should adjust both s and t by adding independent normal rvs to each.

Test your sampler with:

• 
$$f(x,y) = e^{-(x^2+y^2)}$$

• 
$$f(x,y) = \begin{cases} 1 & |x| + |y| < 1 \\ 0 & \text{otherwise} \end{cases}$$

• 
$$f(x,y) = \exp(-0.05(1-x)^2 - 5(y-x^2)^2)$$
 (the Rosenbrock density)

Display the results as scatterplots of sampled points.