

## Math 370 – Take Home Quiz 2

This quiz should take you approximately 25 minutes. You may use your calculator, your book, and your notes, but do not work together and do not get help.

(10) 1. Let  $f(x) = \begin{cases} -1 & \text{if } x \geq 0 \\ 3 & \text{if } x < 0 \end{cases}$

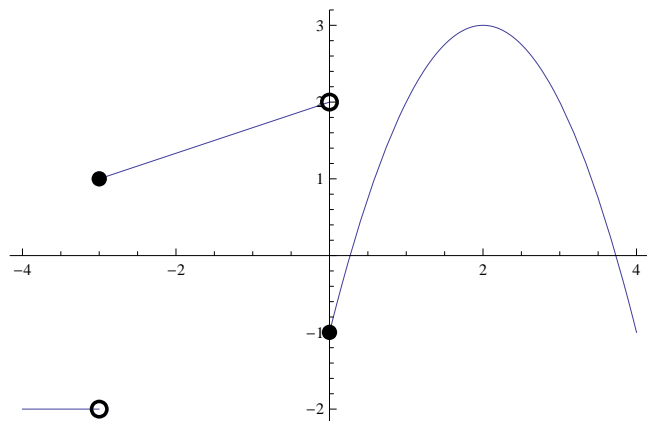
Find the Fourier series for  $f$  on  $[-\pi, \pi]$ . Give at least the first four nonzero terms.

(10) 2. The Fourier sine series for  $f(x) = x/2$  on the interval  $[0, \pi]$  is given by

$$\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \frac{\sin 5x}{5} - \frac{\sin 6x}{6} + \dots$$

Use this series to compute the infinite sum  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$

- (10) 3. The function  $f(x)$  is graphed below, and has a Fourier series on  $[-4, 4]$ .



To what value does the Fourier series converge when:

- (a)  $x = -4$ ?
  - (b)  $x = -3$ ?
  - (c)  $x = 0$ ?
  - (d)  $x = 2$ ?
  - (e)  $x = 4$ ?
- (10) 4. The Fourier sine series for  $f(x) = x/2$  on the interval  $[0, \pi]$  is given by

$$\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \frac{\sin 5x}{5} - \frac{\sin 6x}{6} + \dots$$

- (a) Write a series solution  $u(x, t)$  for the heat equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  with initial condition  $u(x, 0) = x/2$  and ends held at zero temperature:  $u(0, t) = 0$  and  $u(\pi, t) = 0$ .

- (b) Compute the temperature at  $x = \pi/2$  at time  $t = 1/2$  to four decimal places.

(10) 5. Let  $f(x) = \begin{cases} x^2 & \text{if } 0 \leq x < 1 \\ \frac{4-x}{3} & \text{if } 1 \leq x \leq 4 \end{cases}$

(a) Accurately sketch the even periodic extension of  $f(x)$  for  $x \in [-8, 8]$

(b) Accurately sketch the odd periodic extension of  $f(x)$  for  $x \in [-8, 8]$