Math 370 – Sample Vector Calculus Questions

(10) 1. Let $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and let $\varphi(x, y, z) = x + y + z$.

- (a) Which one is defined, $\nabla \mathbf{F}$ or $\nabla \varphi$? Calculate it.
- (b) Which one is defined, $\nabla \cdot \mathbf{F}$ or $\nabla \cdot \varphi$? Calculate it.
- (c) Which one is defined, $\nabla \times \mathbf{F}$ or $\nabla \times \varphi$? Calculate it.

(10) 2. Find a unit normal vector to the surface $x^2 - xy + 4z = 11$ at the point $P = (3, 6, 5)$.

(10) 3. Which of these vector fields are conservative?

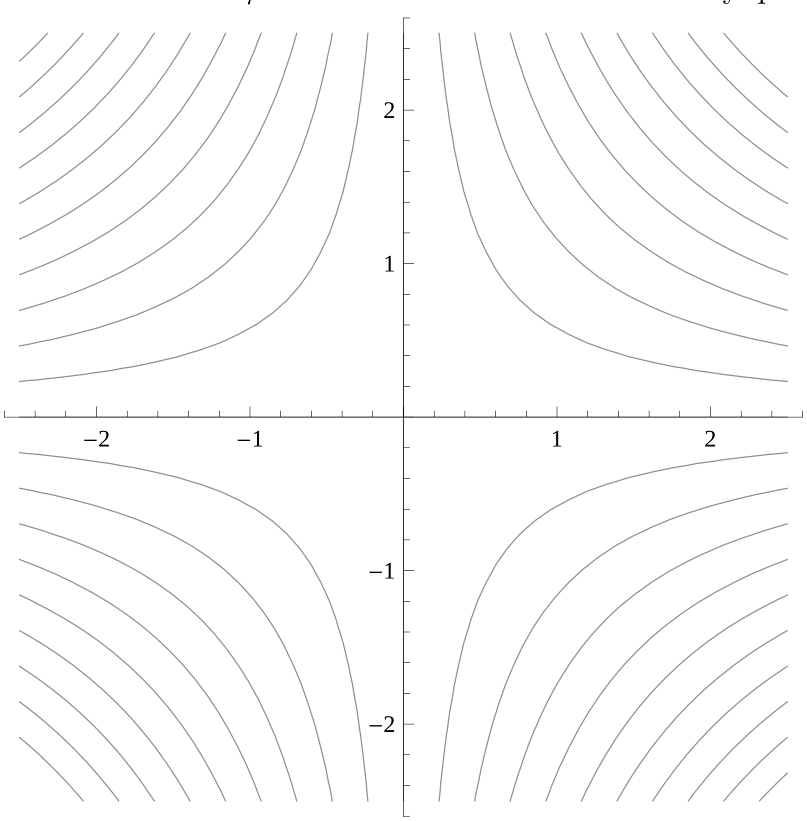
- (a) $\mathbf{F}(x, y) = y^2\mathbf{i} - xy\mathbf{j}$
- (b) $\mathbf{F}(x, y) = \mathbf{i} - \mathbf{j}$
- (c) $\mathbf{F}(x, y) = \frac{1}{x^2+y^2}(-y\mathbf{i} + x\mathbf{j})$
- (d) $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$
- (e) $\mathbf{F}(x, y) = e^x\mathbf{i} + e^y\mathbf{j}$

(10) 4. Find a potential function for $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + \mathbf{k}$.

(10) 5. For $\mathbf{F}(x, y, z)$ a vector field, and $\varphi(x, y, z)$ a scalar function, prove the product rule:

$$\nabla \cdot \varphi\mathbf{F} = \nabla\varphi \cdot \mathbf{F} + \varphi\nabla \cdot \mathbf{F}$$

(10) 6. Level curves for $\varphi(x, y) = \frac{xy}{2}$ are shown below. On the same picture, accurately sketch the vector field $\nabla\varphi$. Plot at least four vectors in every quadrant, plus some on the axes.



(10) 7. Let C be any curve from (-1,4,0) to (3,0,7). Calculate $\int_C 2xdx + zdy + ydz$.

(10) 8. Let $\mathbf{F}(x, y) = (x - 3y, x^2 + 4)$, and let C be the curve that goes once counterclockwise around the rectangle with corners (0,0), (8,0), (8,3), and (0,3). Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(10) 9. For $u \geq 0, v \in [0, 2\pi]$, the parameterization $x = u \cos v, y = u \sin v, z = \frac{u}{3}$ describes an infinite cone. Find a unit normal vector to this cone as a function of u and v.

(10) 10. Find the flux of the vector field $F = e^{-z^2}\mathbf{k}$ through the cone from problem 9.