

Math 370 – Sample Fourier Series Questions

(10) 1. Mark each function as even, odd, or neither:

(a) $\sin(x)$

(a) **Odd**

(b) e^x

(b) **Neither**

(c) $|x - 1|$

(c) **Neither**

(d) x^5

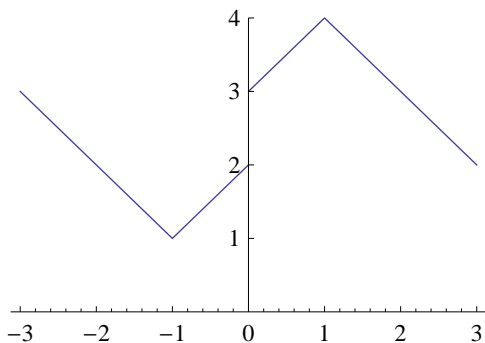
(d) **Odd**

(e) $x^3 \sin(x)$

(e) **Even**

(10) 2. Let $f(x)$ be the function on $[-3, 3]$ which is graphed below. Find the constant term in the Fourier series for f .

Solution: The constant term is $5/2$, the average value of f . The term $a_0 = 5$.



(10) 3. The function f (shown below) is defined on the interval $[-2, 2]$. What value does the Fourier series for f converge to:

(a) When $x = -1$?

(a) **1/2**

(b) When $x = 0$?

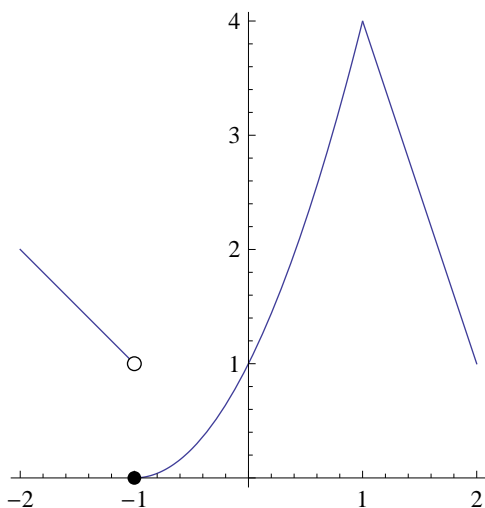
(b) **1**

(c) When $x = 1$?

(c) **4**

(d) When $x = 2$?

(d) **3/2**



- (10) 4. Let $f(x) = |x|$ for $-2 \leq x \leq 2$, and let $g(x) = 4|x| + 3$ for $-2 \leq x \leq 2$. The Fourier series for f is given by

$$f(x) = 1 - \frac{8}{\pi^2} \left(\cos\left(\frac{\pi x}{2}\right) + \frac{1}{9} \cos\left(\frac{3\pi x}{2}\right) + \frac{1}{25} \cos\left(\frac{5\pi x}{2}\right) + \dots \right)$$

What is the Fourier series for g ?

Solution:

$$g(x) = 4f(x) + 3 = 7 - \frac{32}{\pi^2} \left(\cos\left(\frac{\pi x}{2}\right) + \frac{1}{9} \cos\left(\frac{3\pi x}{2}\right) + \frac{1}{25} \cos\left(\frac{5\pi x}{2}\right) + \dots \right)$$

- (10) 5. Let $f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < -\pi/2 \\ 1 & \text{for } -\pi/2 \leq x < \pi/2 \\ 0 & \text{for } \pi/2 < x \leq \pi \end{cases}$.

Find the Fourier series for f on the interval $[-\pi, \pi]$.

Give at least four terms in the series or write it as a summation.

Solution:

$$\frac{1}{2} + \frac{2 \cos(x)}{\pi} - \frac{2 \cos(3x)}{3\pi} + \frac{2 \cos(5x)}{5\pi} - \dots$$