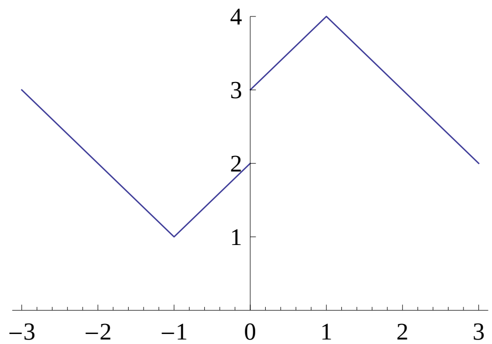
Math 370 – Sample Exam 2

(10) 1. Let $f(x)$ be the function on $[-3, 3]$ which is graphed below. Find the constant term in the Fourier series for f.



(10) 2. Let $f(x) = |x|$ for $-2 \leq x \leq 2$, and let $g(x) = 4|x| + 3$ for $-2 \leq x \leq 2$. The Fourier series for f is given by

$$f(x) = 1 - \frac{8}{\pi^2}\left(\cos\left(\frac{\pi x}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi x}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi x}{2}\right) + \cdots\right)$$

What is the Fourier series for g?

(10) 3. Let $f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < -\pi/2 \\ 1 & \text{for } -\pi/2 \leq x < \pi/2 \\ 0 & \text{for } \pi/2 < x \leq \pi \end{cases}$.

Find the Fourier series for f on the interval $[-\pi, \pi]$.

Give at least four terms in the series or write it as a summation.

(10) 4. The function f (shown below) is defined on the interval $[-2, 2]$. What value does the Fourier series for f converge to:

(a) When $x = -1$?

(a) ____________

(b) When $x = 0$?

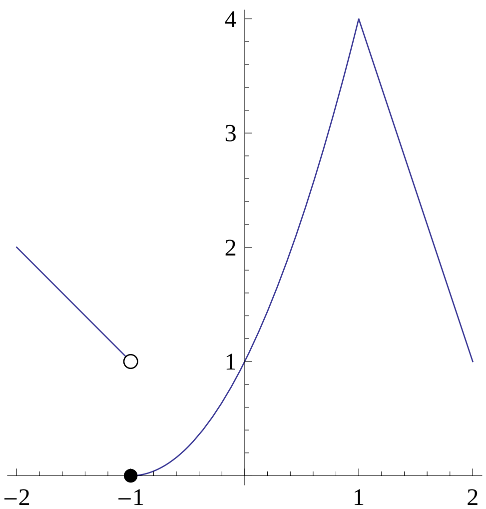
(b) ____________

(c) When $x = 1$?

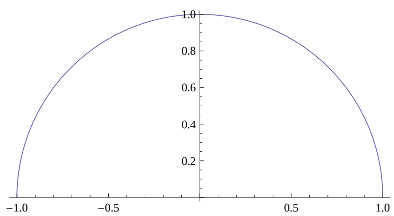
(c) ____________

(d) When $x = 2$?

(d) ____________



(10) 5. Give a clockwise parameterization of the semi-circle shown below:



(10) 6. Let $\mathbf{F}(t) = (2\cos(t), \sin(t))$. Sketch this curve for $0 \leq t \leq 2\pi$. Find the acceleration vector, and find four places where the tangential acceleration is zero.

(10) 7. Let $\mathbf{F}(t) = (t, \cosh(t))$, a catenary curve. Find the curvature of $\mathbf{F}$.

(Hint: The identity $\cosh^2(t) = 1 + \sinh^2(t)$ is helpful).

(10) 8. Let $\mathbf{F} = \mathbf{i} + 3y\mathbf{j} - 3z\mathbf{k}$. Find the streamline for $\mathbf{F}$ through the point (0,2,3).

(10) 9. Let $f(x, y) = y^2 - x$. Accurately draw level curves for f and the vector field ∇f on the same sketch.

(10) 10. Let $\mathbf{P}(t)$ be a parameterized curve with velocity $\mathbf{v}(t)$ and acceleration $\mathbf{a}(t)$. Suppose $m > 0$ is constant. If the curve satisfies $\mathbf{F} = m\mathbf{a}$ for some vector field $\mathbf{F}$ along the curve, prove

$$\frac{d}{dt}(m\mathbf{P} \times \mathbf{v}) = \mathbf{P} \times \mathbf{F}.$$

(The physical meaning of this is that rate of change of angular momentum is the torque.)