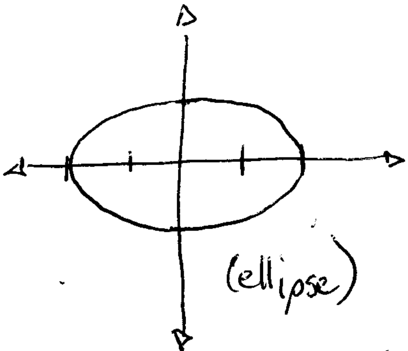
1)



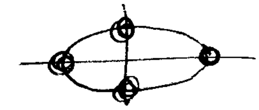
$v(t) = r'(t) = (-2\sin t, \cos t)$

$a(t) = v'(t) = (-2\cos t, -\sin t)$

Accel. is $-2\cos t\,\vec{i} - \sin t\,\vec{j}$

tangential accel. will be 0 when a and v are perpendicular.

So: $a \cdot v = 4\sin t\cos t - \sin t\cos t = 3\sin t\cos t$ which is 0 when $t = 0, \pi/2, \pi, 3\pi/2$ or at these points:



2)
a) $\nabla f = \vec{i} + \vec{j} + \vec{k}$ b) $\nabla \cdot F = 3$ c) $\nabla \times F = 0$

3) Let $\varphi = x^2 - xy + 4z$. $\nabla\varphi = (2x-y, -x, 4)$ $\nabla\varphi(P) = (0, -3, 4)$

$$N = \frac{\nabla\varphi(P)}{\|\nabla\varphi(P)\|} = (0, -3/5, 4/5)$$

4) a) Not b) Conservative c) Not d) Not e) Conservative

In part (c), $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = 0$ except at $(0,0)$ where F is undefined. In fact if $r(t) = (\cos t, \sin t)$ then $\int F \cdot dr = 2\pi$

5) $\varphi = xy + z$

6) Let $F = f\vec{i} + g\vec{j} + h\vec{k}$.

$$\nabla \cdot \varphi F = \frac{\partial}{\partial x}(\varphi f) + \frac{\partial}{\partial y}(\varphi g) + \frac{\partial}{\partial z}(\varphi h) =$$

$$= \frac{\partial\varphi}{\partial x} \cdot f + \varphi\frac{\partial f}{\partial x} + \frac{\partial\varphi}{\partial y} g + \varphi\frac{\partial g}{\partial y} + \frac{\partial\varphi}{\partial z} h + \varphi\frac{\partial h}{\partial z}$$

$$= \frac{\partial\varphi}{\partial x} f + \frac{\partial\varphi}{\partial y} g + \frac{\partial\varphi}{\partial z} h + \varphi\left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}\right)$$

$$= \nabla\varphi \cdot F + \varphi\nabla F$$