Math 370

# Sample Midterm 2

(10 pts) 1. Let $r(t)= (2\cos(t), \sin(t))$.
Sketch this curve for $0 \le t \le 2\pi$.
Find the acceleration vector, and find four places where the tangential acceleration is zero.

(10 pts) 2. Let $F(x,y,z) = (x, y, z)$, and let $f(x, y, z) = x + y + z$.
a. Which one is defined, $\nabla F$ or $\nabla f$? Calculate it.

b. Which one is defined, $\nabla \bullet F$ or $\nabla \bullet f$? Calculate it.

c. Which one is defined, $\nabla \times F$ or $\nabla \times f$? Calculate it.

(10 pts) 3. Find a unit normal vector to the surface $x^2 - xy + 4z = 1$ at the point P = (3,6,5).

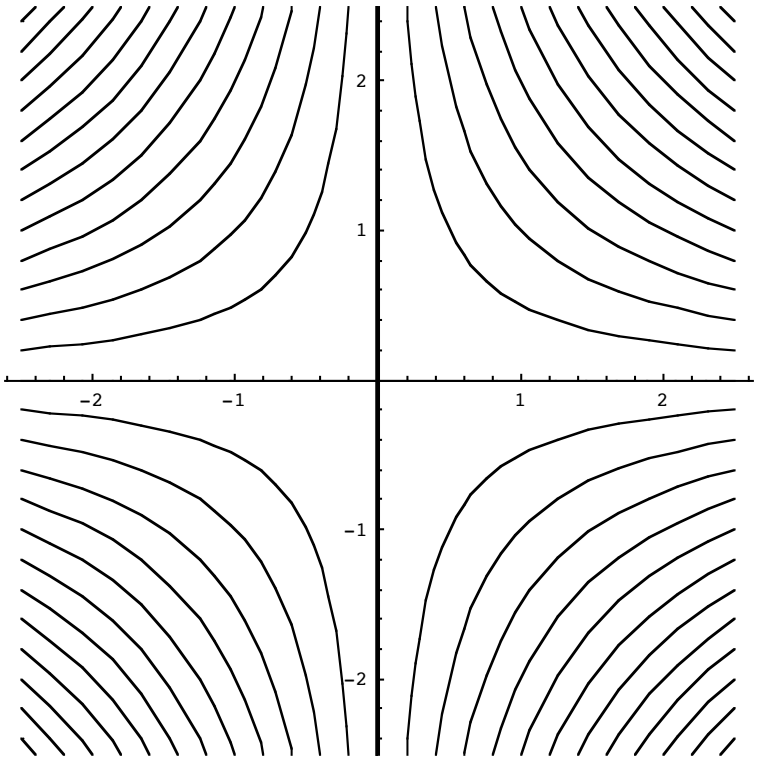
(10 pts) 4. Which of these vector fields are conservative?
a. $F(x,y) = y^2\mathbf{i} - xy\mathbf{j}$
b. $F(x,y) = \mathbf{i} - \mathbf{j}$
c. $F(x,y) = \frac{1}{x^2+y^2}(-y\mathbf{i} + x\mathbf{j})$
d. $F(x,y) = -y\mathbf{i} + x\mathbf{j}$
e. $F(x,y) = e^x\mathbf{i} + e^y\mathbf{j}$

(10 pts) 5. Find a potential function for $F(x,y) = y\mathbf{i} + x\mathbf{j} + \mathbf{k}$.

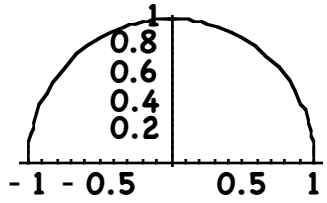
(10 pts) 6. For F a vector field and $\varphi$ a scalar function, prove the product rule:

$$\nabla \bullet \varphi F = \nabla\varphi \bullet F + \varphi \nabla \bullet F$$

(10 pts) 7. Level curves for $f(x,y) = \frac{xy}{2}$ are shown below. On the same picture, accurately sketch the vector field $\mathbf{v} = \nabla f$.
Plot at least four vectors in every quadrant, plus some on the axes.




(10 pts) 8. Give a clockwise parameterization of the semi-circle shown below:




(10 pts) 9. Calculate $\int_{(-1,4,0)}^{(3,0,7)} 2x\,dx + z\,dy + y\,dz$

(10 pts) 10. Let $F(x, y) = (x - 3y, x^2 + 4)$ and let R be the rectangular region with corners (0,0), (8,0), (8,3), and (0,3). Compute the line integral

$\int_C F \bullet dr$

clockwise along the boundary of R.