

6) No. S is not closed under scalar multiplication.

For example, $(1, 0) \in S$ but $-(1, 0) = (-1, 0)$ is not in S .

7) a) Clockwise 90° rotation

b) $N^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ which is counterclockwise 90° rotation.

$$8) \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -2 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{R_3 \rightarrow \frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & \frac{1}{2} \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 + 3R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & \frac{3}{2} \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & \frac{1}{2} \end{array} \right)$$

$$\text{So } A^{-1} = \begin{pmatrix} -2 & 0 & \frac{3}{2} \\ 0 & 1 & 0 \\ -1 & 0 & \frac{1}{2} \end{pmatrix}$$

9) a) $\det A_1 = 1$, $\det A_2 = -2$, $\det A_3 = -6$, $\det A_4 = 24$

$$\det A_5 = 120, \det A_6 = -720$$

b) $\det A_n = (-1)^{\lfloor n/2 \rfloor} n!$ (also ok to say $\pm n!$ with
+ for $n=1, 4, 5, 8, 9, 12, 13, \dots$ and - for $n=2, 3, 6, 7, 10, 11, \dots$)

10) Using $\det(XY) = \det(X)\det(Y)$ and $\det(A^{-1}) = (\det A)^{-1}$

$$\begin{aligned} \det(ABA^{-1}) &= \det(A)\det(BA^{-1}) = \det(A)\det(B)\det(A^{-1}) \\ &= \det(A)\det(B)\det(A)^{-1} = \det B \end{aligned}$$