

MED TERM 1 SAMPLE - SOLUTIONS

①

1) $A = D^T, B = H^T, C = J^T, E = F^T, G = K^T$

2)
$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix} \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1}} \begin{pmatrix} 1 & 2 & 0 \\ 0 & -3 & 3 \\ 0 & -1 & 2 \end{pmatrix} \xrightarrow{R_2 \rightarrow -\frac{1}{3}R_2} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_3 \rightarrow R_3 + R_2} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_3} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3)
$$\begin{pmatrix} 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

$$\xrightarrow{R_1 \rightarrow R_1 - R_3} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

So
$$\begin{aligned} x_1 &= 0 \\ x_2 + x_4 &= 0 \\ x_3 - x_5 &= 0 \end{aligned}$$

x_4, x_5 are free.

General solution is
$$x_4 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

4)
$$X = H^{-1} \begin{pmatrix} 1/4 \\ 0 \\ 0 \\ 1/4 \end{pmatrix} = \begin{pmatrix} 4 - 20 \\ -30 + 240 \\ 60 - 600 \\ -35 + 400 \end{pmatrix} = \begin{pmatrix} -16 \\ 210 \\ -540 \\ 365 \end{pmatrix}$$

5) For example, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix},$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$