Math 370 **Midterm 1** SAMPLE

(10 pts) 1. Match each matrix below with its transpose:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ 2 & 2 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 2 & 2 & 2 \\ 1 & 0 & 2 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 2 \end{pmatrix} \qquad \mathbf{E} = \begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} \qquad \mathbf{F} = \begin{pmatrix} 2 & 2 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{G} = \begin{pmatrix} 0 & 1 \\ 2 & 2 \end{pmatrix}$$

$$\mathbf{H} = \begin{pmatrix} 2 & 1 \\ 2 & 0 \\ 2 & 2 \end{pmatrix} \qquad \mathbf{J} = \begin{pmatrix} 1 & 0 & 2 \end{pmatrix} \qquad \mathbf{K} = \begin{pmatrix} 0 & 2 \\ 1 & 2 \end{pmatrix}$$

(10 pts) 2. Reduce the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$ to reduced row echelon form.

(10 pts) 3. Find all solutions to the homogeneous system of equations:

$$\begin{aligned} x_1 - x_2 + x_3 - x_4 - x_5 &= 0 \\ x_2 + x_4 &= 0 \\ x_3 - x_5 &= 0 \end{aligned}$$

(10 pts) 4. The 4×4 "Hilbert matrix" is $H = \begin{pmatrix} 1 & 1/2 & 1/3 & 1/4 \\ 1/2 & 1/3 & 1/4 & 1/5 \\ 1/3 & 1/4 & 1/5 & 1/6 \\ 1/4 & 1/5 & 1/6 & 1/7 \end{pmatrix}$ and its inverse matrix is $H^{-1} = \begin{pmatrix} 16 & -120 & 240 & -140 \\ -120 & 1200 & -2700 & 1680 \\ 240 & -2700 & 6480 & -4200 \\ -140 & 1680 & -4200 & 2800 \end{pmatrix}$. Solve $Hx = \begin{pmatrix} 1/4 \\ 0 \\ 0 \\ 1/7 \end{pmatrix}$ for x.

(10 pts) 5. Give an example of three vectors which are linearly dependent, but so that no two of them are scalar multiples of each other.

(10 pts) 6. Let $S = \{ (x,y) \mid x \geq 0 \}$, the set of vectors in the plane with non-negative x coordinate. Is S a subspace of $\mathbb{R}^2$? Justify your answer.

(10 pts) 7. Let $N = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

a. Explain geometrically what multiplication by N does to vectors in the plane.

b. Find N^{-1} and explain what multiplication by N^{-1} does to vectors in the plane.

(10 pts) 8. Let $A = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 2 & 0 & -4 \end{pmatrix}$. Find A^{-1}. Show work.

(10 pts) 9. Let $A_1=(1)$, $A_2=\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$, $A_3=\begin{pmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $A_4=\begin{pmatrix} 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ and so on.

a. What are the determinants of A_1, A_2, A_3, A_4, A_5, and A_6?

b. Give a formula (using n) for the determinant of A_n.

(10 pts) 10. Suppose A and B are $n \times n$ matrices, and suppose A is nonsingular. Prove that $\det(ABA^{-1}) = \det(B)$.