Math 370 | Homework 7 | Due Wednesday 10/24

Exercises

Don't hesitate to use a symbolic math system (such as Maple or Wolfram Alpha) to do the integrals required for these problems.

Chapter 13.2 # 13*, 14*, 15*, 20

* For 13-15, first graph $f(x)$. Also, for #14, the second line should read $1 < x \leq \pi$.

Chapter 13.3 # 1**, 3**, 11

** For 1 and 3, first graph the even and odd periodic extensions of $f(x)$.

Problem A: Suppose f is differentiable. Show that when f is even, the derivative f' is odd. Show that when f is odd, the derivative f' is even.

Problem B: Compute (or look up) the Fourier sine and cosine series for $f(x) = x$ on the interval $[0, \pi]$. Show that the derivative of the cosine series for f is the sine series for $f'(x) = 1$ on the interval $[0, \pi]$. On the other hand, show that the derivative of the sine series for f is *not* the cosine series for f'. In fact, the term-by-term derivative of the sine series for f does not converge - graph a large partial sum to see this.

Problem C: Compute (or look up) the Fourier sine series for $f(x) = 1$ on the interval $[0, \pi]$. Use this to write a series solution for the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with initial condition $u(x, 0) = 1$ and ends held at zero temperature: $u(0, t) = 0$ and $u(\pi, t) = 0$.

Graph the third partial sum of this series for $t = .01$, $t = .1$, and $t = 1$.

(This material is covered in Chapter 17.2.1 of O'Neil)