NOTE: This is a revised version of homework 6 as of October 12. Please discard the old version. The main changes are that problems 13, 14, 15 and 20 are dropped, while 9, 12 and Z are added.

Exercises

Don't hesitate to use a symbolic math system (such as Maple or Wolfram Alpha) to do the integrals required for these problems.

Chapter 13.2 # 1, 2, 6, 9, 10, 12

Problem Z: For each function, decide if it is even, odd, or neither:

    (a) $\sin(x)$

    (b) e^x

    (c) $|x - 1|$

    (d) x^5

    (e) $x^3 \sin(x)$

Problem A: Consider the function $f(x) = x^3 + 1$, $-1 \le x \le 1$. Find the constant term a_0 of its Fourier series on $[-1, 1]$, and explain why all the cosine terms will vanish.

Problem B: Let $f(x) = \frac{x^2}{2}$. Find the Fourier series for f on the interval $[-\pi, \pi]$. Plug in $x = \pi$ and use the result to compute:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

This was first proved by Euler in 1734.

Problem C: Find the Fourier series for $\cos^3(x)$ on the interval $[-\pi, \pi]$ and use that to prove the triple angle identity $\cos(3x) = 4\cos^3(x) - 3\cos(x)$.