

Exercises

Chapter 7.6 # 1, 4, 9 (beware that the answers in the back of the book are wrong for many of the 7.6 problems, including #9)

Chapter 7.7 # 1, 2, 3, 9, 15

Chapter 8.1 # 1, 3

Problem A: Let \mathbf{A} be the matrix from Section 7.7 Exercise 9. Solve $\mathbf{A}\mathbf{v} = \mathbf{b}$ when \mathbf{b} is each of the following vectors:

$$\mathbf{b}_1 = \begin{pmatrix} 4 \\ 0 \\ 12 \end{pmatrix}; \quad \mathbf{b}_2 = \begin{pmatrix} 0 \\ 12 \\ 0 \end{pmatrix}; \quad \mathbf{b}_3 = \begin{pmatrix} -12 \\ 24 \\ -12 \end{pmatrix}$$

Problem B: Let $\mathbf{R}_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ be a rotation matrix. Show that $\mathbf{R}_\theta^{-1} = \mathbf{R}_{-\theta}$.

Problem C: A matrix \mathbf{A} is called *orthogonal* if $\mathbf{A}^t = \mathbf{A}^{-1}$. Show that if \mathbf{A} is orthogonal then $\mathbf{A}\mathbf{A}^t = \mathbf{I}$ and $\mathbf{A}^t\mathbf{A} = \mathbf{I}$. Show that if \mathbf{A} is orthogonal then \mathbf{A}^t is orthogonal.

Problem D: Show that \mathbf{R}_θ is orthogonal.

Problem E: Suppose \mathbf{A} and \mathbf{B} are the same size, and suppose that both \mathbf{A} and \mathbf{B} are orthogonal. Show that $\mathbf{A}\mathbf{B}$ is orthogonal.

Problem F: Suppose \mathbf{A} and \mathbf{B} are the same size. If \mathbf{A} is symmetric and \mathbf{B} is orthogonal, show that $\mathbf{B}\mathbf{A}\mathbf{B}^{-1}$ is symmetric.

Problem G: Suppose \mathbf{A} is orthogonal. Show that the column vectors of \mathbf{A} are orthogonal to each other and that each column vector of \mathbf{A} has magnitude 1.

Hint: The product of two matrices has entries which are dot products of rows of one matrix with columns of the other.

Problem H: Suppose \mathbf{A} is orthogonal. Show that multiplication by \mathbf{A} preserves lengths. That is, for any vector \mathbf{v} , show that $\|\mathbf{A}\mathbf{v}\| = \|\mathbf{v}\|$.

Hint: For a column vector \mathbf{w} , you can compute $\mathbf{w} \cdot \mathbf{w} = \mathbf{w}^t\mathbf{w}$.