Exercises

Chapter 7.1 # 3,6,14,15,17,19,21,23

Problem A: Give an example of two 2x2 matrices $\mathbf{A}$ and $\mathbf{B}$ so that $\mathbf{AB} = \mathbf{0}$ but $\mathbf{BA} \neq \mathbf{0}$.

Problem B: Give an example of nonzero 2x2 matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ so that $\mathbf{AC} = \mathbf{BC}$ but $\mathbf{A} \neq \mathbf{B}$.

Problem C: A matrix is *symmetric* if it is equal to its transpose. Give an example of a 2x2 symmetric matrix. Give an example of a 3x3 symmetric matrix. Can you give an example of a 2x3 symmetric matrix?

Problem D: Let $\mathbf{A}$ be any $n \times m$ matrix.

(a) What size matrix is $\mathbf{A}^t\mathbf{A}$? Show that it is symmetric.

(b) What size matrix is $\mathbf{A}\mathbf{A}^t$? Show that it is symmetric.

Problem E: Suppose $\mathbf{A}$ is a square matrix. Show that $\mathbf{A} + \mathbf{A}^t$ is symmetric. Why did $\mathbf{A}$ need to be square?

Problem F: Suppose $\mathbf{A}$ and $\mathbf{B}$ are symmetric. Give an example to show that $\mathbf{AB}$ may not be symmetric. Show that if $\mathbf{AB} = \mathbf{BA}$ then $\mathbf{AB}$ *is* symmetric.

Problem G: Let $\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. For example, $\mathbf{R}_{30^\circ} = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$ What is $\mathbf{R}_{45^\circ}$?

Problem H: (a) Let $\mathbf{u} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, and $\mathbf{w} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$. Sketch these three vectors.

(b) Sketch the vectors $\mathbf{R}_{45^\circ}\mathbf{u}$, $\mathbf{R}_{45^\circ}\mathbf{v}$, $\mathbf{R}_{45^\circ}\mathbf{w}$.

(c) What effect did multiplication by $\mathbf{R}_{45^\circ}$ have on the three vectors?

(d) What effect does multiplication by $\mathbf{R}_{120^\circ}$ have on the three vectors?

(e) Explain why $\mathbf{R}_\theta$ is called a"rotation matrix".

Problem I: (a) Show that $\mathbf{R}_\alpha\mathbf{R}_\beta = \mathbf{R}_{\alpha+\beta}$. (Hint: use a sum-of-angles identity.)

(b) What does the equation of part (a) tell you about rotations?

Problem J: The matrix $\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is called a *projection matrix*. Describe, geometrically, the effect of multiplying a 2-vector by this matrix.