Exercises

Chapter 11.5 # 1, 2, 7, 11, 13

For problems A, B, C, let $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be the radial vector field, and let $\rho = ||\mathbf{R}||$ be the scalar function "distance from the origin".

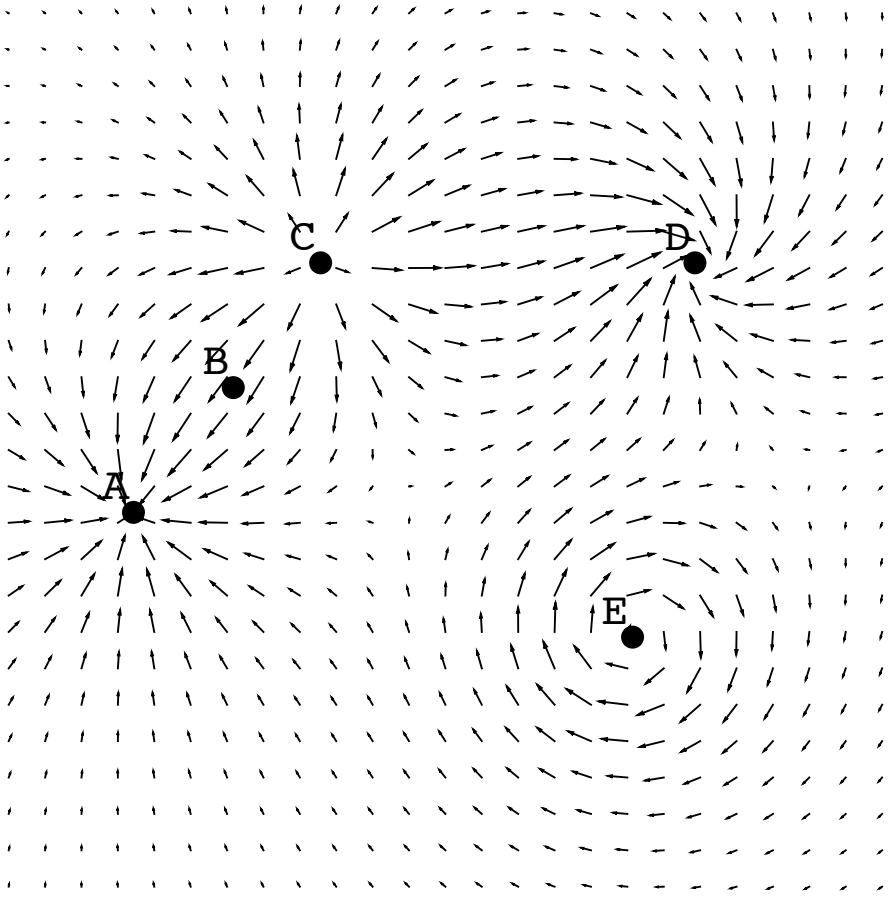
Problem A: Show that $\nabla \rho^n = n\rho^{n-2}\mathbf{R}$ for $n = 1, 2, \ldots$

Problem B: Let $\mathbf{A}$ be a constant vector. Compute

1. $\nabla \cdot (\mathbf{R} - \mathbf{A})$
2. $\nabla \times (\mathbf{R} - \mathbf{A})$

Problem C: Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable real valued function of one variable. Show that $\nabla \times f(\rho)\mathbf{R} = \mathbf{0}$. (Hint: Use the product rule from problem 13).

Problem D: For each point A-E in the vector field $\mathbf{F}$ shown below, decide 1) if $\nabla \cdot \mathbf{F}$ is positive, negative, or zero, and 2) if the $\mathbf{k}$ component of $\nabla \times \mathbf{F}$ is positive, negative, or zero.



Problem E: Prove conclusion (2) of O'Neil Theorem 11.3, that is, show $\nabla \cdot \nabla \times \mathbf{F} = 0$.